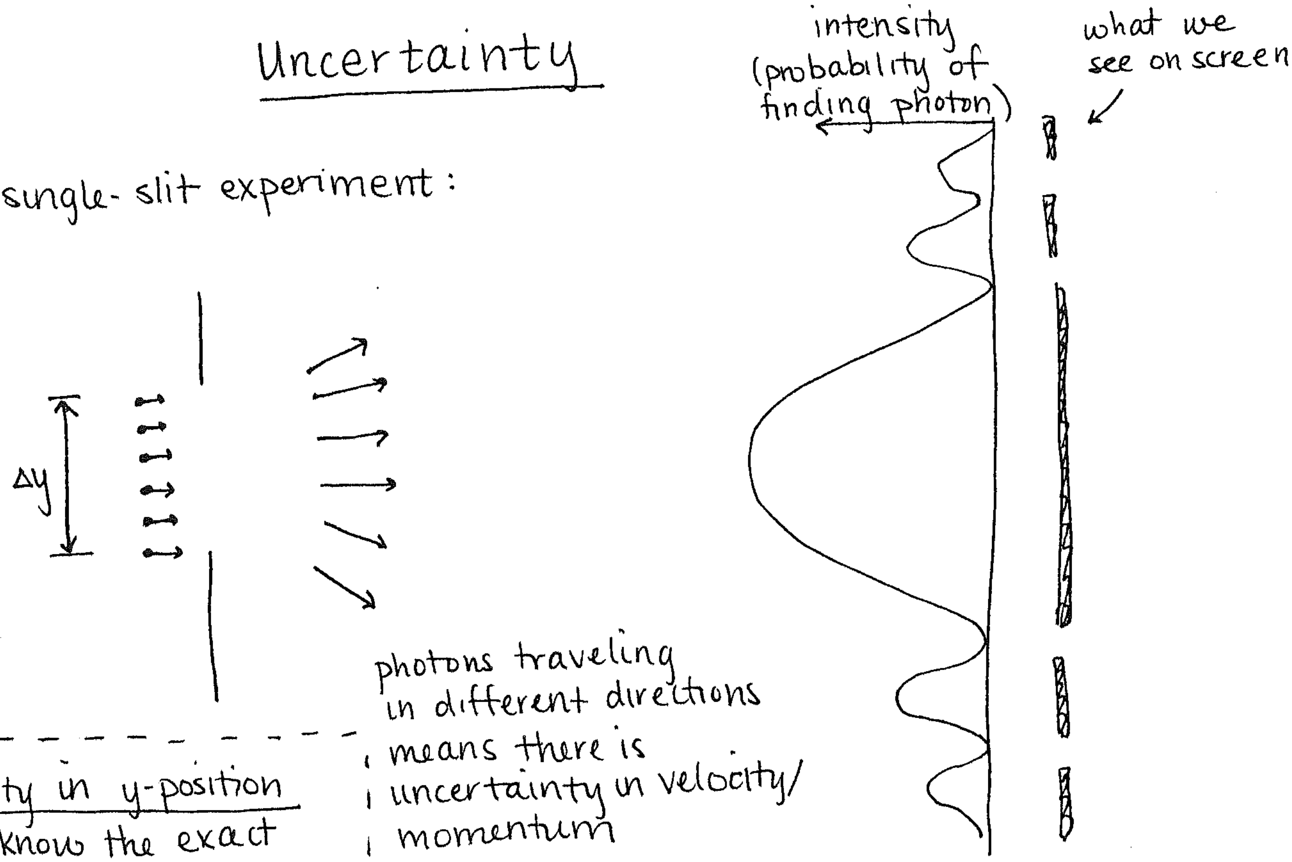
Uncertainty

Recall the single-slit experiment:



photons traveling in different directions means there is uncertainty in velocity/momentum

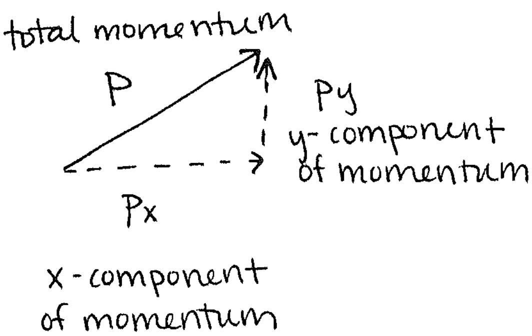
Δy = uncertainty in y-position

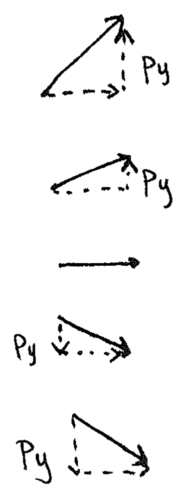
we don't know the exact y-position of the photons as they pass through the slit, so there is uncertainty in y (there is a range of different y-values, and the size of that range is Δy)

since the photons land at many different places on the screen, we know that the photons are traveling in different directions as they leave the slit. This means that the velocity vector is pointing in different directions for different photons

ΔP_y = uncertainty in y-momentum

(remember: $\Delta P_y = m \Delta V_y$)





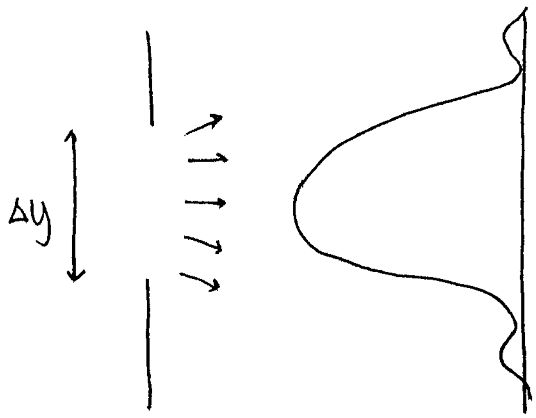
large, positive P_y

zero P_y

large, negative

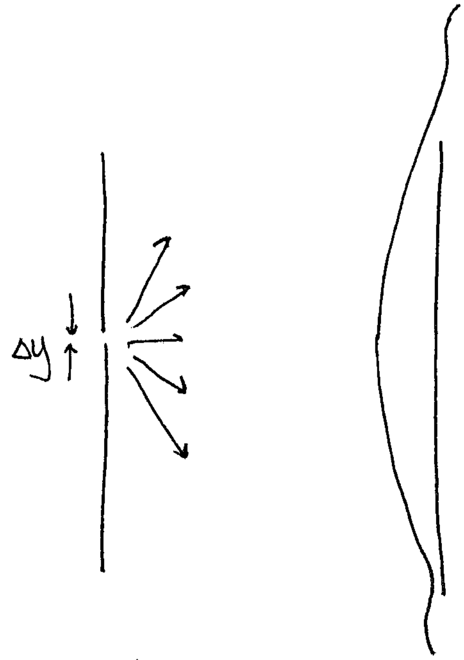
different values of P_y mean that we are uncertain about the value of P_y

Uncertainty



large Δy
(big slit)

small Δp_y
(localized / not spread out on screen)



small Δy
(small slit)

large Δp_y
(very spread out on screen)

We see that Δy and Δp_y are linked together, such that when Δy gets smaller, Δp_y gets bigger (and vice-versa). Δy and Δp_y are examples of <u>conjugate variables</u>. More examples are:

cartesian (x, y, z) coordinates	Δx	Δp_x	position / momentum uncertainty
	Δy	Δp_y	
	Δz	Δp_z	
spherical (r, θ, ϕ) coordinates	Δr	Δp_r	
	$\Delta \theta$	Δp_θ	
	$\Delta \phi$	Δp_ϕ	
	Δt	ΔE	energy / time uncertainty

Uncertainty

There is a set of relationships that tie 2 conjugate variables together:

Heisenberg Uncertainty Principle:

(if Δx is large, Δp_x is small, and vice-versa)

$$\Delta x \, \Delta p_x \geq \frac{\hbar}{2}$$

$$\Delta y \, \Delta p_y \geq \frac{\hbar}{2}$$

$$\Delta z \, \Delta p_z \geq \frac{\hbar}{2}$$

position / momentum uncertainty

$$\Delta t \, \Delta E \geq \frac{\hbar}{2}$$

(if Δt is large, ΔE is small, and vice-versa)

energy / time uncertainty

These relationships place a <u>limit</u> on how much we can know: the more we know (the more certain we are) about one variable, the less we know (the more uncertain we are) about the other.

The <u>very best</u> that we can do is

$$\Delta x \, \Delta p_x = \frac{\hbar}{2} \qquad \Delta t \, \Delta E = \frac{\hbar}{2}$$

minimum uncertainty = very best we can do

This tells us that:

if $\Delta x = 0$ (we know the exact value of x) $\Longrightarrow$ $\Delta p_x = \infty$ (we have absolutely <u>no idea</u> what the value of p_x is!)

★ note: $\hbar$ is just a fancy name/symbol for a constant that appears over and over again in QM. Just as the inch is a unit that measures length, $\hbar$ is a unit that measures angular momentum

Uncertainty

Let's consider some examples:

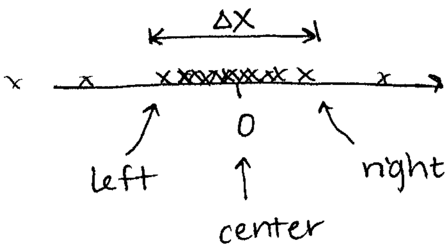
Grain of Sand

If the position of a grain of sand has uncertainty Δx, this means that if I measure the position x many, many times, I will find a range of values of x

Position:

$\Delta x =$ size of range of values of x (standard deviation in x)

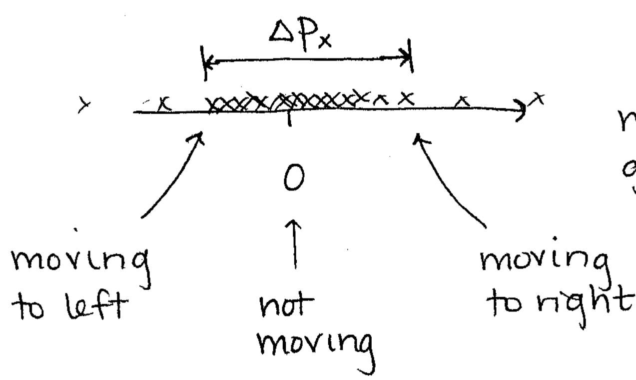
many different measurements



$x =$ position of grain of sand

Momentum:

$\Delta p_x =$ size of range of p_x values (standard deviation in p)



$p_x =$ momentum of grain of sand

A grain of sand is large (compared to the size of an atom), and therefore the overall uncertainty is small. In the classical world (the world we see everyday), we can ignore uncertainty (it is still there, but it is very very small). However, in the quantum world, uncertainty plays a <u>huge</u> role, and we cannot ignore it.