How do we find the wavefunction?

shape of wave → probability at every point

1. whats happening at edges of system
   Boundary Conditions

2. governing equation

Classical

Diffusion Eq

- diffusion of molecules/density ⟷
- ~~them~~ molecules spread out over time

Wave Eq

- waves of water, pressure
- water/pressure waves stayed localized as they traveled

Quantum

Schrodinger Eq

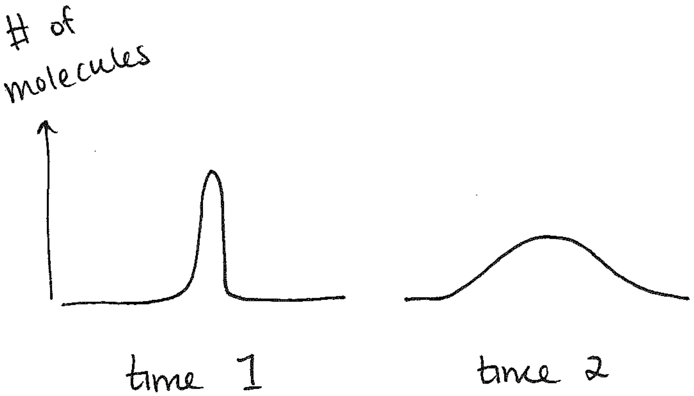
- diffusion of probability
- probability spreads out over time
- when we "turn off" time, ~~(steady~~ (stationary states / standing prob. waves)

looks like waves

KG, Dirac, Rel. QM Eq

Governing Equation

Classical World



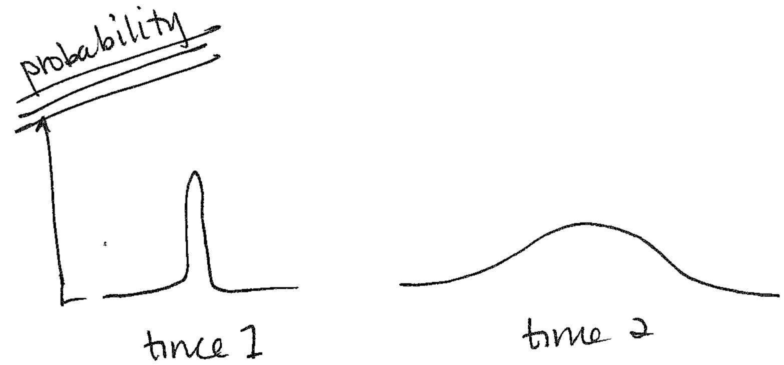
Diffusion Equation:
describes how <u>density</u> spreads out over time

curvature $\longleftrightarrow$ velocity

(of density wave ~~profile~~ function) (of density wave ~~function~~)

$$\underbrace{\boxed{\frac{\partial^2}{\partial x^2}}}_{\text{curvature of density}}(\rho) \longleftrightarrow \underbrace{\boxed{\frac{\partial}{\partial t}}}_{\text{velocity of density}}(\rho)$$

Quantum World



Schrodingers Equation
describes how <u>probability</u> spreads out over time

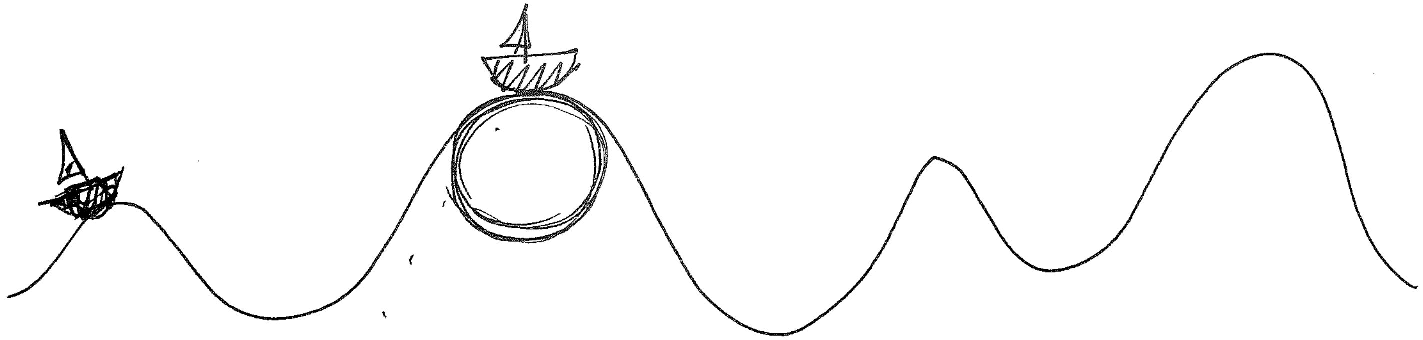
curvature $\longleftrightarrow$ velocity

(of ~~wave function~~ prob. wave) (of prob. wave)

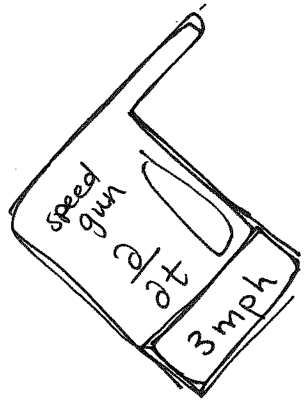
$$\frac{\partial^2}{\partial x^2}(\psi) \longleftrightarrow \frac{\partial}{\partial t}(\psi)$$

<u>different</u> than velocity of particle

Example (Derivatives)

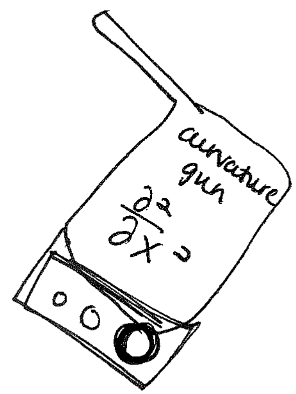


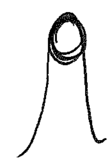
~~Let's say~~
Want.



speed
"operator"

operate on boat to get speed





curvature
"operator"

operate on boat to get curvature

You have two measurements = (3mph, ◎) and from these measurements,
you want to find the ~~position~~ height of boat

how fast the boat is moving up + down

$\frac{\partial}{\partial t}$(height) $\longleftrightarrow$ $\frac{\partial^2}{\partial x^2}$(height of)

speed gun   curvature gun

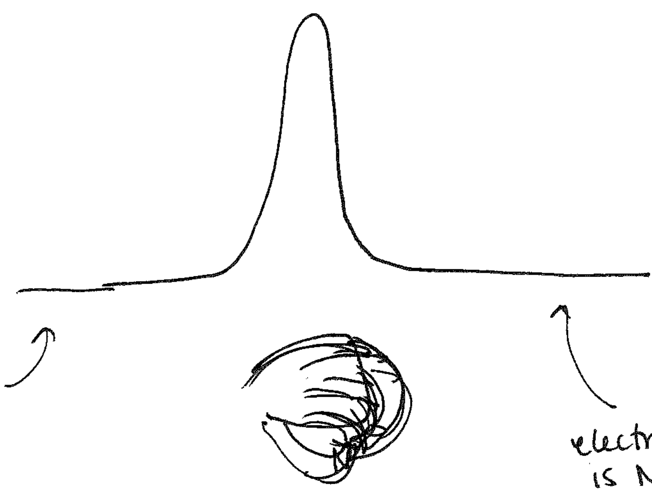
how much the boat is tipping from front to back

What does this mean?

$$\boxed{-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} = i\hbar\frac{\partial \psi}{\partial t}}$$

hold an electron
in our hand

let electron go →
see what happens

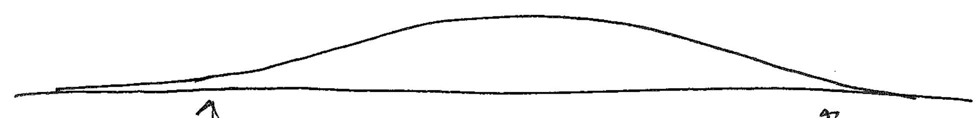


not here

electron is NOT here

⟵⟶
electron is localized

$\boxed{\text{probability is localized}}$



can now find
(w/ small prob)
the electron out
here

$\boxed{\text{probability is spread out}}$

Closer Look at S.E.

$$\underbrace{-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi(x,t)}{\partial x^2}}_{\text{curvature of probability wave}} = \underbrace{i\hbar\frac{\partial \Psi(x,t)}{\partial t}}_{\text{velocity of probability wave}}$$

[same as $a\frac{\partial^2}{\partial x^2}\left(\text{height of }\Psi\right) = b\frac{\partial}{\partial t}\left(\text{height of boat}\right)$]

different than velocity of particle

What is this equation telling us physically? about the particle?

conjugate variables $x \leftrightarrow p$

what causes changes in position? momentum p

what causes changes in time? energy E

$t \leftrightarrow E$

(specifically: $\hat{p} = i\hbar\frac{\partial}{\partial x}$ $\hat{E} = i\hbar\frac{\partial}{\partial t}$)

can rewrite S.E. in terms of p and E:

$$\underbrace{\frac{p^2}{2m}}_{\text{KE}}\Psi(x,t) = \underbrace{E}_{\text{total energy}}\Psi(x,t)$$

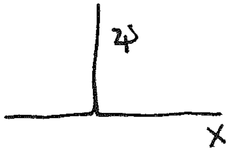
KE: kinetic energy

much nicer looking!

S.E. is just a statement of conservation of energy!

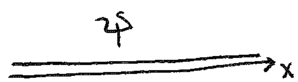
Turn off time

Know x exactly



know nothing about p (all values of p are equally probable)

Know nothing about x (all values of x are equally probable → looks same at all values of x



Know p exactly

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} = i\hbar\frac{\partial \psi}{\partial t}$$

$$\updownarrow \qquad\qquad \updownarrow$$

$$\frac{p^2}{2m}\psi = E\psi$$

know t exactly

know nothing about E (looks same at all values of E)

know nothing about t (looks same at all values of t)

know E exactly

What do we know about a wave that looks same at all values of t?

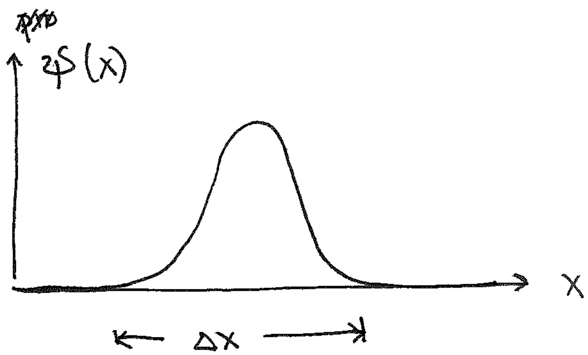
→ standing wave / stationary state

"Turn off" time ⟶ look at constant E

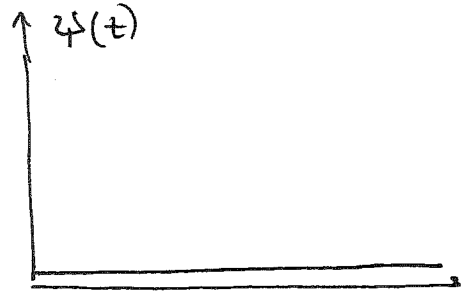
Assume that we don't know p, x exactly (there is uncertainty in both x and p)

TISE

Position

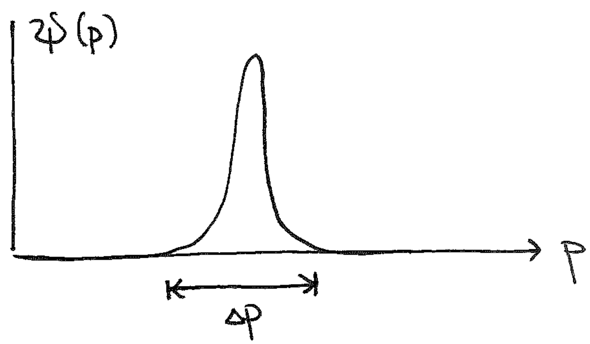


time



$\Delta t = \infty$ (looks same at all values of t)

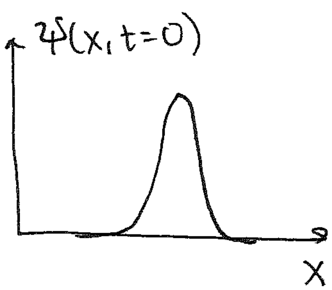
Momentum:

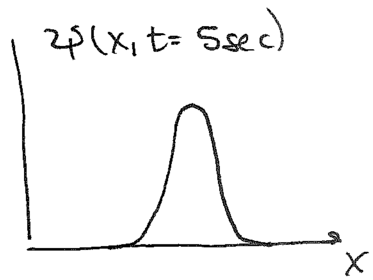


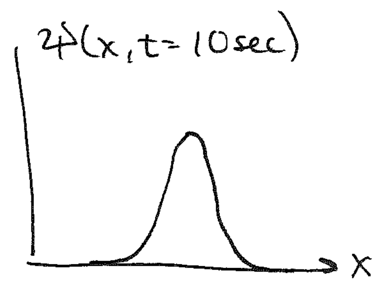
Energy

$\Delta E = 0$ (<u>no</u> uncertainty)

Example:







shape of ψ doesn't change in time!